Escape of photons from infinite cylinders

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Escape of photons from infinite cylinders

Abstract. It is found that photons emitted from the surface of an infinitely long cylindrical mass distribution and moving in a plane perpendicular to its axis are allowed to escape to infinity only when the mass per unit length of the cylinder is below a certain critical limit. In all other cases they will be recaptured by the cylinder, with the exception of those moving radially.

In a recent communication Synge (1966) has shown that for 'gravitationally intense stars' only those photons which are emitted within a slender critical cone can escape to infinity. But in the limit when the surface of the star approaches the Schwarzschild radius only the radially moving photons can escape. In a corresponding problem involving a cylindrically symmetric mass distribution it is found that, for photons moving in a plane normal to the cylindrical axis, there exists a critical value for the mass per unit length of the cylinder below which the photons escape to infinity for all angles of emergence. In other cases they have to turn back somewhere in their courses, the only exception being those moving radially.

The well-known metric (Marder 1958) in vacuum outside an indefinitely long static cylinder is given by

$$ds^2 = r^{2C}dt^2 - r^{2(1-C)}d\phi^2 - A^2r^{-2C(1-C)}(dr^2 + dz^2) \tag{1}$$

where C and A are constants. $\frac{1}{2}C$ is interpreted as the mass per unit length of the cylinder. Now the equations for the null geodesics along which the z coordinate is fixed may be expressed by

$$r^{2C}\dot{t}^2 - r^{2(1-C)}\dot{\phi}^2 - A^2r^{-2C(1-C)}\dot{r}^2 = 0 \tag{2}$$

$$\ddot{t} + \frac{2C}{r}\dot{t}\dot{r} = 0 \tag{3}$$

$$\ddot{\phi} + \frac{2(1-C)}{r}\dot{r}\dot{\phi} = 0 \tag{4}$$

where the dots denote differentiation with respect to some affine parameter. The equations (3) and (4) after integration give

$$\dot{t} = \alpha r^{-2C} \qquad \text{and} \qquad \dot{\phi} = \beta r^{-2(1-C)}$$

where α and β are constants. After combining the above two relations one can directly obtain

$$\dot{t} = \frac{m}{r^{2(2C-1)}}\dot{\phi} \tag{5}$$

where $m = \alpha/\beta$ is an arbitrary constant.

Substituting (5) in the equation (2) it is easy to obtain the relation

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{1}{A^2} r^{2(1-C^2)}\{r^{2(1-2C)}m^2 - 1\}. \tag{6}$$

Now if ψ is the angle of inclination made by the light ray with the radial direction, and if we assume that the spatial components of unit vectors along them are (dr, $d\phi$, 0) and (dr', 0, 0) respectively, then

$$\cos\psi = \frac{g_{11}drdr'}{(g_{11}dr'^2)^{1/2}(g_{11}dr^2 + g_{33}d\phi^2)^{1/2}}$$

$$= \left\{1 + \frac{g_{33}}{g_{11}}\left(\frac{d\phi}{dr}\right)^2\right\}^{-1/2} \tag{7}$$

In the above we use symbols 1, 2, 3, 4 for r, z, ϕ and t respectively. The equation (7) gives

$$\cot^2\psi = \frac{A^2 r^{-2C(1-C)}}{r^{2(1-C)}}\left(\frac{dr}{d\phi}\right)^2 = (m^2 r^{2(1-2C)} - 1). \tag{8}$$

Equation (8) can be written also in the form

$$m^2 = r^{2(2C-1)}\operatorname{cosec}^2\psi = r_0^{2(2C-1)}\operatorname{cosec}^2\psi_0$$

r_0 and ψ_0 being the values of r and ψ on the boundary at the instant of emission. In view of equation (6) it is evident that, since $(dr/d\phi)^2$ is essentially positive, m^2 must be larger than $r^{2(2C-1)}$. The value of C ($\frac{1}{2}C$ giving the mass per unit length) is quite significant in the present discussion, for, when $C \leqslant \frac{1}{2}$, $2C-1 \leqslant 0$ and once the light ray is emitted in any of the directions from the surface it can escape to infinity because m^2 is then always greater than $r^{2(2C-1)}$. In the case $C > \frac{1}{2}$, $2C-1$ is always positive and though the ray is initially emitted from the surface it must turn back later since $r^{2(2C-1)}$ gradually increases and finally becomes equal to m^2. As an only exception the ray can traverse an infinite distance provided m^2 is infinite which means ψ_0 is equal to zero. Thus, excepting the radially moving light ray, all other rays are recaptured when $C > \frac{1}{2}$. The importance of the present discussion is that unlike the spherically symmetric case the radius of the cylinder does not play any decisive role in determining whether photons will escape to infinity or will be recaptured by the source.

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